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Linear stress relations for a metal matrix composite sandwich beam with any core material

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In this paper, it is shown that shear stresses are developed in the interface between the facing material and the core of a sandwich beam. The sandwich beam is composed of a core of any suitable material sandwiched between an upper unreinforced metal facing and a bottom facing made from metal matrix composite (MMC) material. The shear stress is shown to be a consequence of the differences in the core and facing elastic moduli. The magnitude of the shear stress increases as the core stiffness is made to diminish. The shear stress can exceed the bond strength between facing and core, resulting in delamination. Consequently, structural materials using this type of construction and particularly flexural experiments should contain a relatively stiff core. The magnitude of the facing stresses is shown to be relatively insensitive to the assumption or neglect of these shear stresses. In the worst case considered, neglecting the interfacial shear stresses results in an overestimation of the compressive and tensile stresses by less than 5%.

1. Introduction

Elsewhere, Schoutens [1] developed a theory to obtain stress, strain, and elastic parameters for a metal matrix composite (MMC) sandwich beam with any core material. In that analysis, the stress distribution was assumed continuous across the sandwich interfaces above and below the neutral axis but discontinuous at the neutral axis as shown in Fig. 1a. This assumption was convenient for the analysis, but somewhat higher stress values in the sandwich facing materials are introduced than when the shear stress across the interfaces between facing material and the core is considered. In this paper, the theory presented earlier [1] is extended to account for this effect. The problem is important in assessing flexural test data of composite materials.

2. Theory

The basic assumption is that the strain is everywhere continuous and linear across the entire beam cross-section as shown by the dashed line in Fig. 1b. The core material has an elastic modulus E_l, the upper facing material has an elastic modulus E_m and cross-sectional area A_u, and the bottom facing is an MMC of elastic modulus E_c and cross-sectional area A_b. The upper and bottom facing thicknesses are t_u and t_b, respectively, and the beam has a total height $H = t_u + t_c + t_b$, where t_c is the core thickness, and a width W. The cross-sectional area of the core above the neutral axis is A_{um}, and below it is A_{bm}. A plus sign (+) is used to designate the side of the interface inside the upper or bottom facing, and a minus sign (−) designates the side of the interface inside the core (Fig. 1b). It is further assumed that the interfaces are perfect bonds. A bending moment is introduced in the beam in such a manner as to keep the surface strains within the elastic region and maintain any beam cross-section planar. Then, the stresses on either side of the upper interface are

$$\sigma_{ic+} = \varepsilon_+ E_m \quad (1)$$

$$\sigma_{ic-} = \varepsilon_- E_l \quad (2)$$

where the subscript c stands for compression. But at the interface, $\varepsilon_+ = \varepsilon_- = \varepsilon$, so that one obtains from Equations 1 and 2

$$\frac{\sigma_{ic+}}{\sigma_{ic-}} = \frac{E_m}{E_l} \quad (3)$$

which shows the presence of a discontinuity and, therefore, a shear stress at the interface of magnitude

$$\tau_u = \sigma_{ic+} - \sigma_{ic-} = \varepsilon(E_m - E_l) \quad (4)$$

if $E_m > E_l$. If $E_m = E_l$, then obviously $\tau_u \equiv 0$, as it should be, and the sign of the shear stress is reversed when $E_m < E_l$. The same analysis applied to the bottom interface yields

$$\frac{\sigma_{it+}}{\sigma_{it-}} = \frac{E_c}{E_l} \quad (5)$$

giving a shear stress at that interface of magnitude

$$\tau_b = \sigma_{it+} - \sigma_{it-} = \varepsilon(E_c - E_l) \quad (6)$$

where $\tau_b > 0$ if $E_c > E_l$, and vice versa, and $\tau_b \equiv 0$ when $E_c = E_l$. Note that if $E_c > E_m > E_l$, then the neutral axis is below the geometric centre of the beam, which results in $\sigma_{ic+} > \sigma_{it+}$ and $\sigma_{ic-} > \sigma_{it-}$. The position of the neutral axis for this kind of sandwich beam with any core was calculated by Schoutens [1] to be independent of interfacial shear stresses.

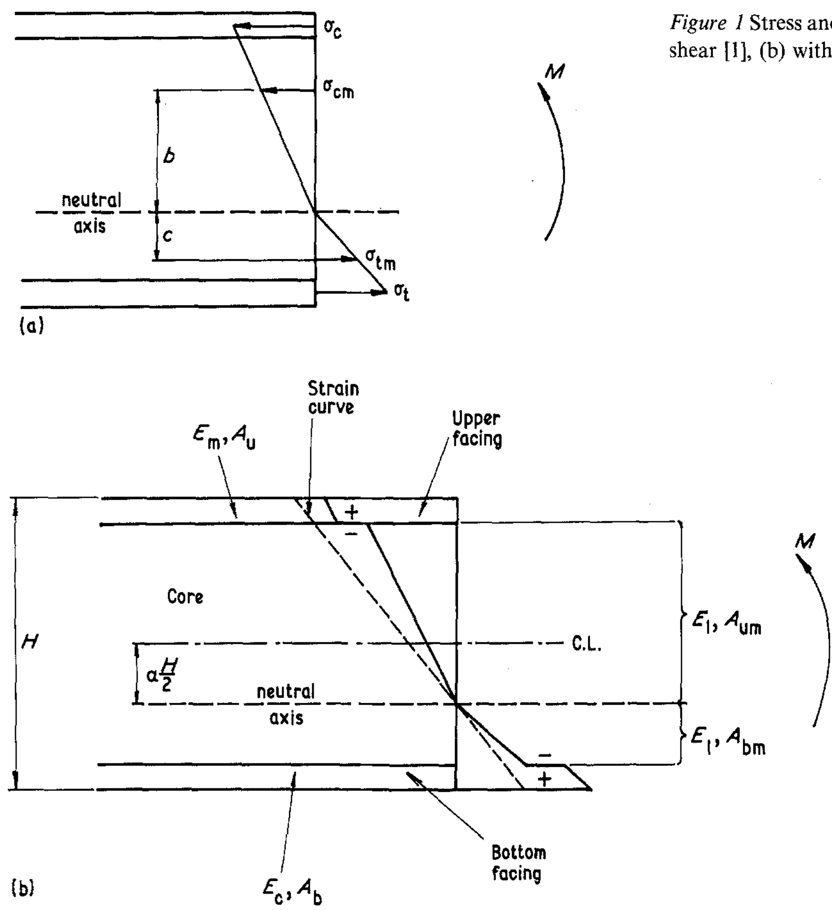

Figure 1 Stress and strain diagrams in a sandwich beam, (a) without shear [1], (b) with interface shear.

Fig. 2 shows the stress components in the core and the facing materials with their positions from the neutral axis given by [1]

$$a = [(1 + \alpha)H - t_u]/2 \quad (7)$$

$$b = [(1 + \alpha)H - 2t_u]/3 \quad (8)$$

$$c = [(1 - \alpha)H - 2t_b]/3 \quad (9)$$

$$d = [(1 - \alpha)H - t_b]/2 \quad (10)$$

$$e = [(1 + \alpha)H - 2t_u]/2 \quad (11)$$

$$f = [(1 - \alpha)H - 2t_b]/2 \quad (12)$$

where α is a dimensionless fraction used to locate the neutral axis. Thus, $X_n = \alpha H/2$. The neutral axis will be below the geometric axis if the composite material is on the tensile side of the beam. If the composite is on the compressive side of the beam, the neutral axis may not be exactly symmetrical on the other side of the geometric axis because the compressive stress–strain behaviour in MMCs, particularly matrices with continuous unidirectional reinforcing fibres, is not exactly the same as the tensile stress–strain diagram. This is due to the fibre compressive microbuckling that occurs in some MMCs, even within the composite elastic strain limit due to residual stresses and other effects.

Calculation of the stresses at the centre of the facing material thickness (σ_c at $t_u/2$ or σ_t at $t_b/2$) requires the equation for the sum of the bending moment, M, or [1]

$$M = W(\sigma_c t_u a + \tfrac{3}{2}\sigma_{cm} b^2 + \tfrac{3}{2}\sigma_{tm} c^2 + \sigma_t t_b d) \quad (13)$$

where σ_{cm} and σ_{tm} are shown in Fig. 2. Now, σ_{cm} and

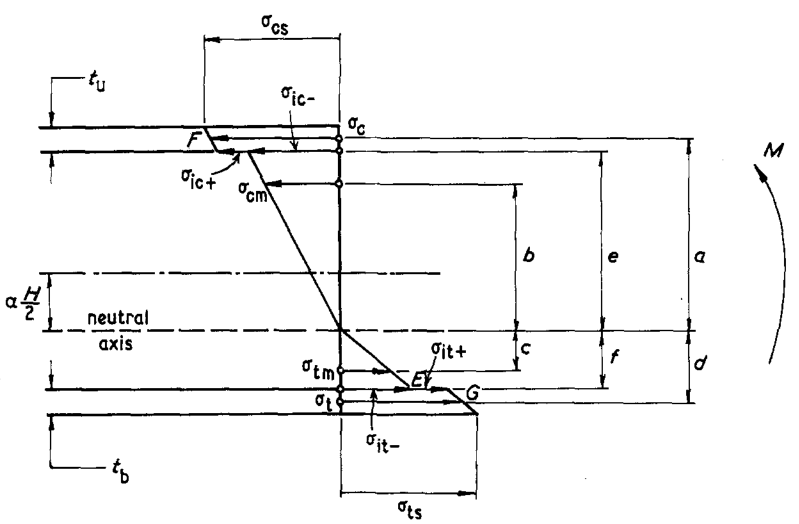

Figure 2 Distribution of stresses in a sandwich beam.

$\sigma_{\rm tm}$ must be related to σ_c and σ_t. As shown in Fig. 2, from the similarity of triangles, $\sigma_{\rm ic-}/e = \sigma_{\rm cm}/b$, so that

$$\sigma_{\rm ic-} = \frac{e}{b}\sigma_{\rm cm}, \quad (14)$$

and equating this relation to Equation 3 and solving for $\sigma_{\rm cm}$ gives

$$\sigma_{\rm cm} = \sigma_{\rm ic+}\frac{b}{e}\frac{E_l}{E_{\rm m}} \quad (15)$$

From the similarity of triangles (Fig. 2),

$$\sigma_{\rm ic+} = \frac{e}{a}\sigma_c, \quad (16)$$

and substituting Equation 16 into 15 gives

$$\sigma_{\rm cm} = \frac{b}{a}\left(\frac{E_l}{E_{\rm m}}\right)\sigma_c. \quad (17)$$

An identical analysis for the lower face gives

$$\sigma_{\rm tm} = \frac{c}{d}\left(\frac{E_l}{E_c}\right)\sigma_t. \quad (18)$$

Equations 17 and 18 relate the core stress above and below the neutral axis to the stresses in the centre of the facing material, accounting for the stress discontinuity across the interfaces. Substituting Equations 17 and 18 into Equation 13 and solving for σ_c gives the compressive stress as a function of the tensile stress in the composite, or

$$\sigma_c = -\frac{d\left[t_b + \frac{3}{2}\left(\frac{E_l}{E_c}\right)\frac{c^3}{d^2}\right]}{a\left[t_u + \frac{3}{2}\left(\frac{E_l}{E_{\rm m}}\right)\frac{b^3}{a^2}\right]}\sigma_t = -A_s\sigma_t \quad (19)$$

which accounts for the shear across the interface, as can be seen by the presence of the ratio of elastic moduli. Note that the coefficient A_s is also dependent upon the value of α through Equations 7 through 10.

When the core material has a very low stiffness, or $E_l \ll E_{\rm m}$ and $E_l \ll E_c$, Equation 19 reduces to

$$\sigma_c \simeq -\frac{d}{a}\sigma_t, \quad (20)$$

assuming that $t_u = t_b$. However, for a very weak core, the danger exists that the facing can delaminate from the core since the shear stress τ_u or τ_b can exceed the bonding strength. Thus, as E_l decreases, τ_u and τ_b increase. This is one reason for having a relatively strong core in a sandwich beam flexural test.

Previously, Schoutens [1] obtained a relationship between σ_c and σ_t in which the shear stress was not considered, or

$$\sigma_c = -\frac{d\left(t_b + \frac{3}{2}\frac{c^3}{d^2}\right)}{a\left(t_u + \frac{3}{2}\frac{b^3}{a^2}\right)}\sigma_t = -A\sigma_t \quad \text{(no shear)} \quad (21)$$

which shows the absence of the effects of the ratio of elastic moduli of the material composing the beam.

Equations 19 and 21 can now be compared to estimate the importance of neglecting the shear at the interfaces. First, it can be assumed for convenience that $t_u \simeq t_b = t$ and, moreover, that

$$\frac{3}{2}\frac{c^3}{d^2} \simeq \frac{3}{2}\frac{b^3}{a^2} \simeq \frac{H}{9} \quad (22)$$

when using Equations 7 to 10. Letting $\Delta_c = E_l/E_c$ and $\Delta_{\rm m} = E_l/E_{\rm m}$,

$$\frac{A_s}{A} \simeq \frac{1 + \Delta_c H/9t}{1 + \Delta_{\rm m} H/9t}. \quad (23)$$

An upper and a lower limit for this ratio can be estimated when it is assumed that $E_c \simeq 3E_{\rm m}$ for the upper limit, and when the core is a metal foam of small cell dimensions [2], $E_l/E_{\rm m} \simeq 1/17$. Thus, $\Delta_{\rm m} \simeq 1/17$ and $\Delta_c \simeq 1/51$, so that Equation 23 yields $A_s/A = 0.959$ if $t \simeq H/10$. This means that neglecting shear at the interface results in an overestimation of the compressive stress in the upper face of about 4% and an overestimation of the tensile stress in the bottom face of the same amount. When considering a weak core, or assuming $E_l/E_{\rm m} \simeq 1/100$ and $E_c \simeq 3E_{\rm m}$, then $\Delta_c \simeq 1/300$ and $\Delta_{\rm m} \simeq 1/100$, so that $A_s/A \simeq 0.992$. This results in an overestimation of the compressive stress of 0.7% and an overestimation of the tensile stress of the same magnitude.

The tensile stress in terms of the bending moment can now be calculated by substituting Equations 17 through 19 into the moment Equation 13 and solving for the tensile stress. This gives

$$\sigma_t = 2adM\Bigg/\left(W\left\{a\left[2t_b d^2 + 3\left(\frac{E_l}{E_c}\right)C^3\right] - d\left[2t_u a^2 + 3\left(\frac{E_l}{E_{\rm m}}\right)b^3\right]\right\}\right) \quad (24)$$

The bending moment can be obtained directly from surface strain measurements. Elsewhere [1], it was shown that the surface compressive and tensile strains $\varepsilon_{\rm cu}$ and $\varepsilon_{\rm tb}$, respectively, can be related to the radius of curvature of the beam, or

$$\varepsilon_{\rm cu} = (1 + \alpha)\frac{H}{2r} \quad (25a)$$

$$\varepsilon_{\rm tb} = (1 - \alpha)\frac{H}{2r}, \quad (25b)$$

and taking either of Equation 25, solving for r and substituting into $M = EI/r$ for a beam in pure bending, gives

$$M = -\frac{2EI}{(1 + \alpha)H}\varepsilon_{\rm cu} \quad (26a)$$

or

$$M = \frac{2EI}{(1 - \alpha)H}\varepsilon_{\rm tb} \quad (26b)$$

where E is the modulus of the beam and I is its moment of inertia with respect to the neutral axis. To arrive at either of Equations 26, the beam span in bending is assumed of such magnitude that the shear contribution is assumed negligible. Zweben [3] has shown that good results are obtained when

$$\frac{\delta_s}{\delta_b} = 1.2\left(\frac{E}{G}\right)\left(\frac{H}{L}\right)^2 \ll 1 \qquad (27)$$

where δ_s and δ_b are the contributions to the beam deflections due to shear and bending, respectively, G is the shear modulus, L is the beam span between supports, and H is the beam height. If condition 27 is satisfied, then shear contribution can be neglected. In testing for material properties in flexure, particularly composite properties, it is desirable to satisfy condition 27; otherwise, large differences result between flexure and tensile test results [3].

The surface stresses are the highest in the beam. These stresses can be determined, with little error, from the values given by Equations 21 and 24 assuming that the change in the stress value across the facing material thickness is uniform and linear. Therefore,

$$\frac{\sigma_{cs}}{a + t_u/2} = \frac{\sigma_c}{a} \qquad (28a)$$

$$\frac{\sigma_{ts}}{d + t_b/2} = \frac{\sigma_t}{d} \qquad (28b)$$

from which

$$\sigma_{cs} = \left[1 + \frac{t_u}{2a}\right]\sigma_c = \varepsilon_{cu}E \qquad (29)$$

$$\sigma_{ts} = \left[1 + \frac{t_b}{2d}\right]\sigma_t = \varepsilon_{tb}E \qquad (30)$$

where σ_{cs} and σ_{ts} are the surface compressive and tensile stresses, respectively, σ_c and σ_t are given by Equations 21 and 24, respectively, and E is the sandwich beam elastic modulus.

3. Conclusions

The interfaces between the upper and bottom facing material of a sandwich beam and the core develop shear stresses during flexure. These shear stresses are the result of the difference in the elastic moduli of the facing and core. In this analysis, it is assumed that the sandwich beam is composed of a metal upper face, a core, and a bottom face made from high-stiffness, high-strength MMC material. During bending, the neutral axis is below the geometric beam centre line if the composite facing is in tension, and vice versa. The magnitudes of the stresses induced in the facing materials do not depend significantly on whether or not one assumes the presence or absence of this shear component, the error being at most less than 5%. However, the presence of the shear stresses at the interface cannot be neglected because their magnitudes grow as the core material stiffness is made to diminish relative to the facing material stiffnesses. Consequently, for a weak core, the shear stress magnitude can exceed the interface bonding strength. This is one reason for having a relatively stiff core in a sandwich beam.

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